

Infinite Horizon Optimal Control Problems – Lebesgue and Riemann improper Integrals

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Abstract In this paper we consider infinite horizon optimal control problems of the following type:

$$J(x, u) = \int_0^{\infty} f_0(t, x(t), u(t)) dt \longrightarrow \text{Min!} \quad (1)$$

subject to $x \in W_{p,\nu}^{1,n}(0, \infty)$, $u \in L_{p,\nu}^r(0, \infty)$, satisfying a. e. on $(0, \infty)$
state equations

$$x'(t) = g(t, x(t), u(t)),$$

control restrictions

$$u(t) \in U, \quad U \in \text{Comp}(R^r) \setminus \{\emptyset\},$$

initial conditions

$$x(0) = x^0.$$

Different criteria of optimality are known for such problems, e. g. the overtaking criterion of v. Weizsäcker (1965), the catching up criterion of Gale (1967) and the sporadically catching up criterion of Galkin (1974). Corresponding to these criteria we prove sufficient conditions for local optimality. Here local optimality is understood in the sense weighted Sobolev spaces $W_{p,\nu}^{1,n}(0, \infty)$, [S], i.e.

$$\|x - x^*\|_{W_{p,\nu}^{1,n}(0, \infty)}^2 := \int_0^{\infty} \{\|x(t) - x^*(t)\| + \|(x'(t) - x^{*'}(t))\|^p \nu(t)\} dt < \epsilon, \quad \epsilon > 0$$

with a density function ν , with $0 < \nu(t) < \infty$, a.e. on $(0, \infty)$ and $\int_0^{\infty} \nu(t) dt < \infty$.

Some aspects concerning the integral in (1) are essential. We allow both Lebesgue and improper Riemann integrals to appear in integrand of the objective. It can happen, that the integral in (1) as Lebesgue integral does not exist and at the same time the Riemann integral is conditionally convergent. We give applications where the improper Riemann integral represents the adequate formulation of the problem and discuss analytic properties of the functional.

Keywords: Infinite horizon optimal control, Weighted sobolev Spaces, Lebesgue and Riemann improper integral.

References

- Carlson, D.A. and A.B. Haurie, A. Leizarowitz, Infinite Horizon Optimal Control, Springer-Verlag, New York, Berlin, 1991.
- Kufner, A., Weighted Sobolev Spaces, John Wiley and Sons, Chichester-New York-Brisbane-Toronto-Singapore, 1985.
- Pickenhain S. and V. Lykina, Sufficiency conditions for infinite horizon optimal control problems, Proceedings of the 12th French-German-Spanish Conference on Optimization held in Avignon, September 20-24, 2004; to appear in Lectures Notes in Economics and Mathematical Systems, Springer-Verlag.
- Rosen, R.: Dynamic System Theory in Biology, Wiley-Interscience, 1970. Please, insert your references