Neural-Network-Based Models Ensemble for Identification in Distributed-Parameter Systems with Application to Elastic Materials Modeling

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Clamped plate description

$$\rho\frac{\partial^2 y(x,t)}{\partial t^2}+\kappa\nabla^4 y(x,t)=p(x,t), \tag{1}$$

where $y(x,t)$ – transverse displacement, $p(x,t)$ – pressure field, x – spatial point, t – time, and

$$\kappa=\frac{Ed^3}{12(1-\nu^2)}, \tag{2}$$

where $E=7.11\cdot 10^{10}$ – the modulus of elasticity, $\nu=0.3$ – the Poisson's ratio, and $\rho=2700$ – mass density.

Domain partitioning

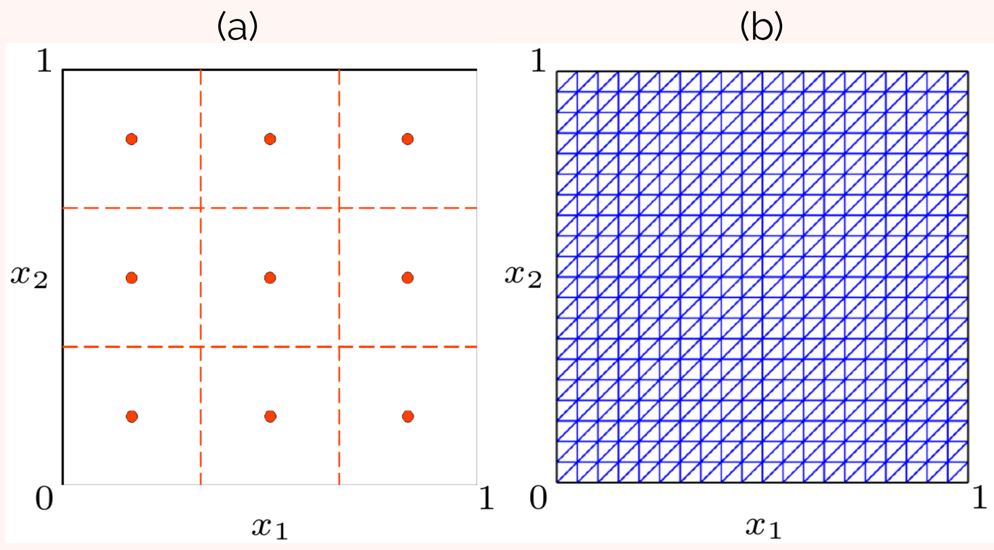

Figure 1. Partition of the clamped plate: our approach for $R=3$ (a), Finite Element Method for 441 nodes (b)

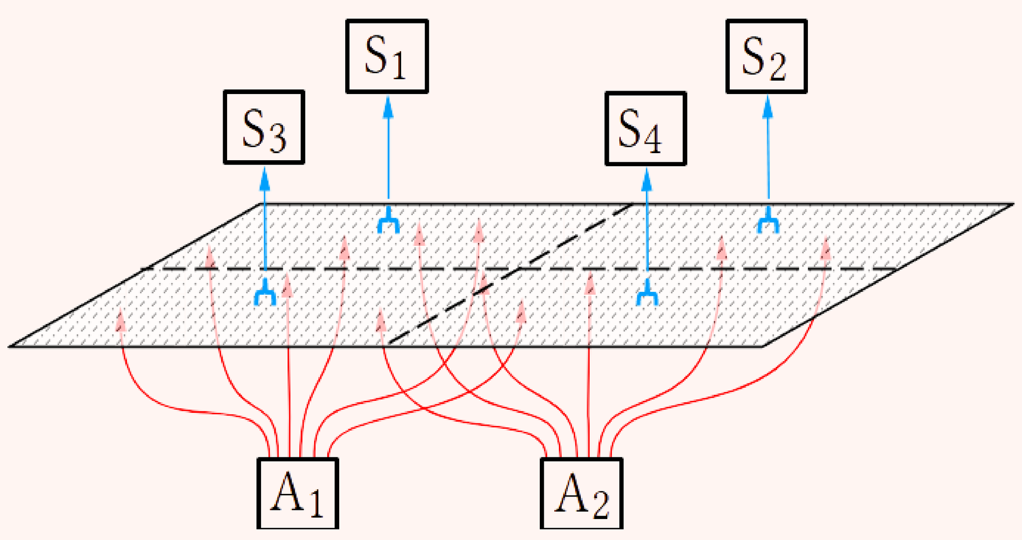

Figure 2. Actuating and sensing of the clamped plate

State-space neural network

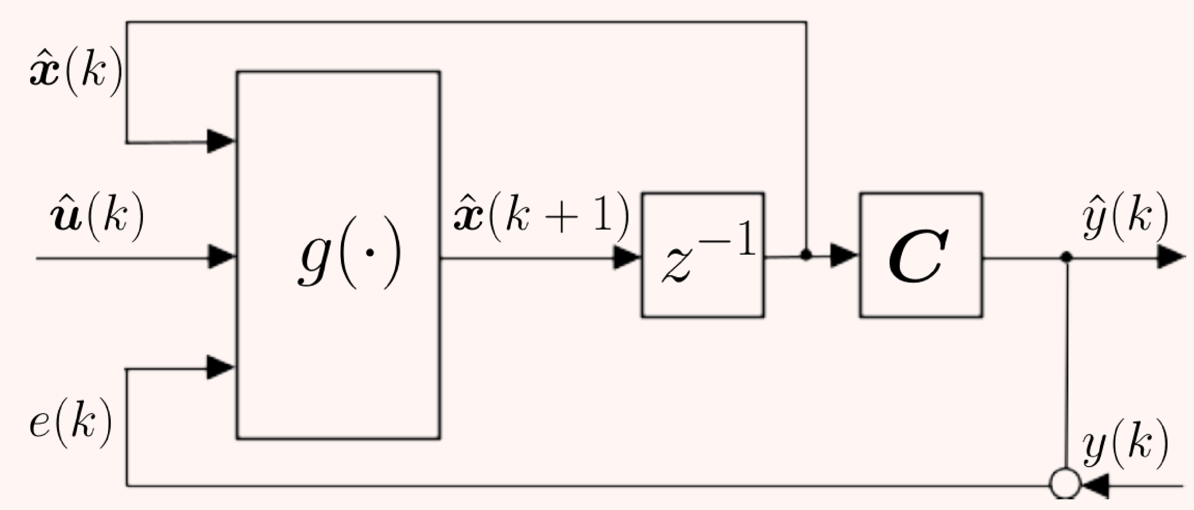

Figure 3. Neural network architecture

$\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ – adaptable weight matrices

Description

$$\hat{\boldsymbol{x}}(k+1)=g(\boldsymbol{\varphi}(k)), \tag{3}$$
$$y(k)=\boldsymbol{C}\hat{\boldsymbol{x}}(k), \tag{4}$$

where

$$\boldsymbol{\varphi}(k)=[\hat{\boldsymbol{x}}(k),\boldsymbol{u}(k),e(k)],$$
$$e(k)=y(k)-\hat{y}(k),$$

and

$$g(\boldsymbol{\varphi}(k))=\boldsymbol{\alpha}\tanh(\boldsymbol{\beta}\boldsymbol{\varphi}(k)). \tag{5}$$

Ensemble design and training

Excitation – two actuators

$$p_1(x_1,x_2,t)=20te^{-25\left((x_1-0.25)^2+(x_2-0.25)^2\right)},$$
$$p_2(x_1,x_2,t)=20(15-t)e^{-50\left((x_1-0.75)^2+(x_2-0.75)^2\right)}$$

where $t=0,\dots,10$, with sampling time $T_s=0.1s$

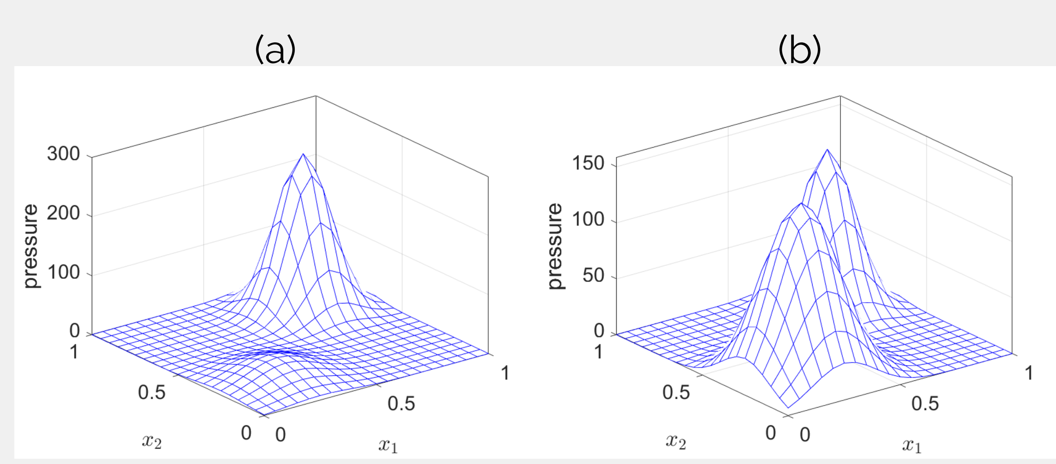

Figure 4. Evolution of the actuation at 2nd second (a) and 8th second (b)

Input data

1. inputs located in the centres of each partition (Fig. 1a)
2. inputs located as in finite element method (Fig. 1b)

Output data

- Plate displacement $y(x_1,x_2,t)$ measured at the centers of partitions
- Outputs number = R^2

Ensemble selection

Table 1. Specification of neural models for $R=3$

Model number	Hidden layer	Model order	Scaling factor
1, 2, 4–6, 8	5H	2	1000
9	5H	2	5000
7	5H	2	10000

Table 2. Specification of neural models for $R=4$

Model number	Hidden layer	Model order	Scaling factor
1-3,5-7,9-12,15-16	5H	2	5000
4	5H	2	10000
8	5L	2	10000
13	5H	2	20000
14	5L	2	20000

Table 3. Specification of neural models for $R=5$

Model number	Hidden layer	Model order	Scaling factor
1–4, 6–20, 22–24	5H	2	5000
25	7H	3	10000
21	5H	3	40000
5	7L	3	40000

Displacement estimation

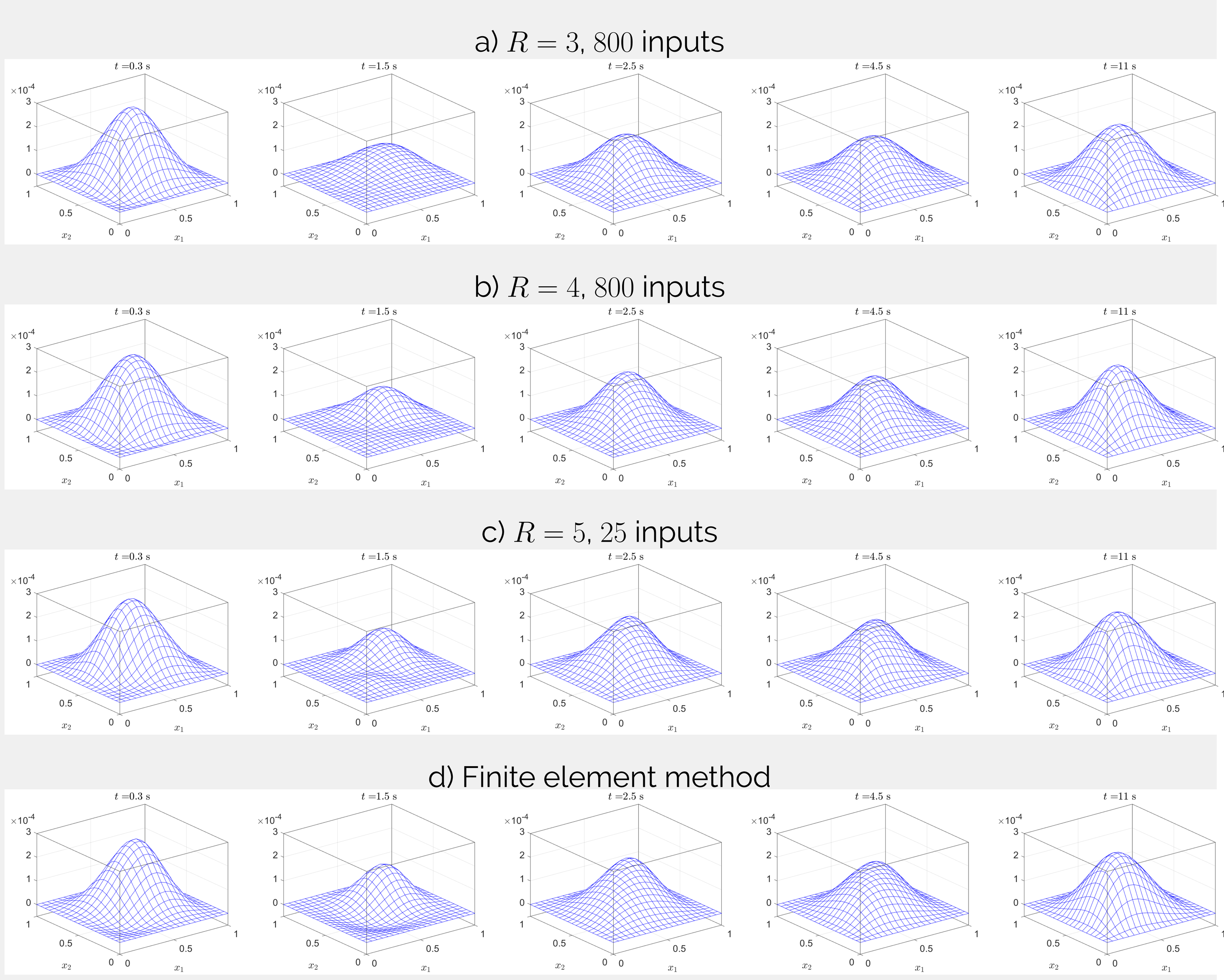

Figure 5. Modeling results for selected time instances: ensembles for $R=3$ and 800 inputs (a); ensembles for $R=4$ and 800 inputs (b); ensembles for $R=5$ and 25 inputs (c); finite element method (d)

Modeling results

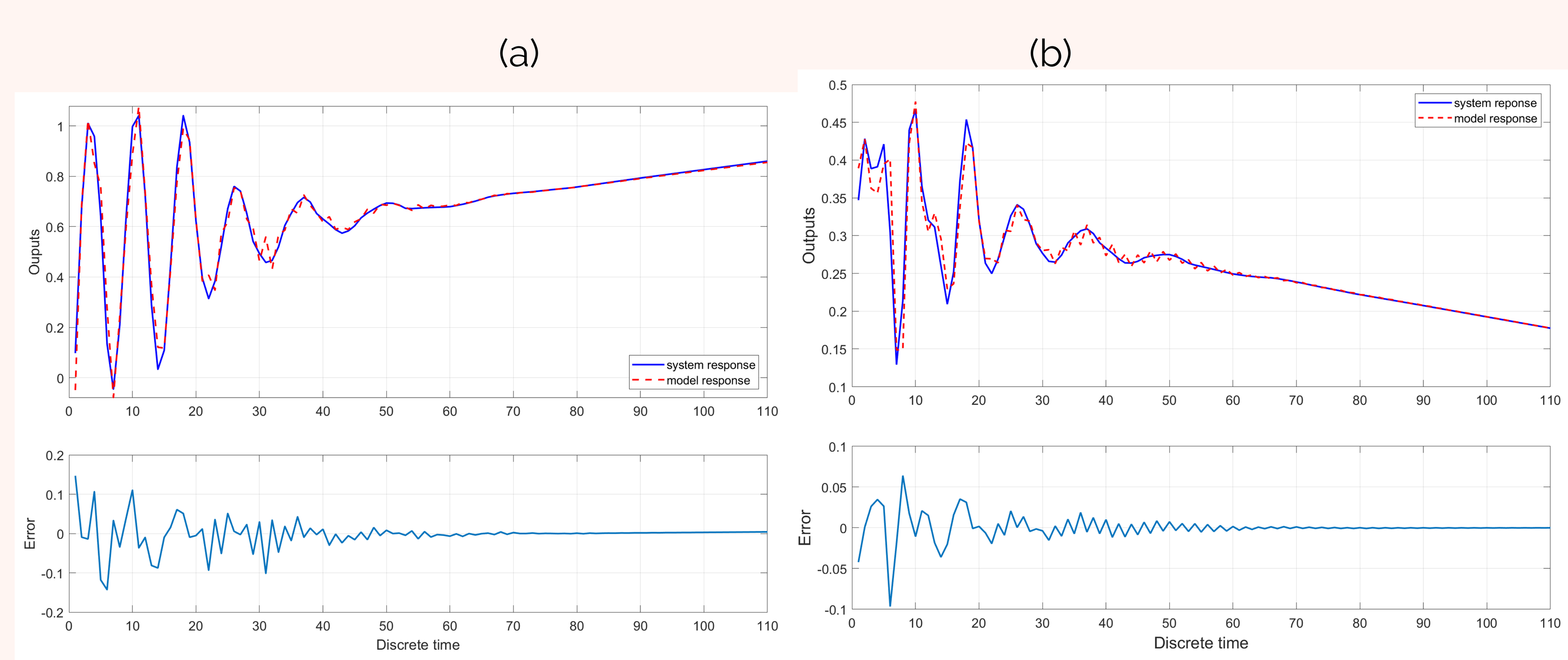

Figure 6. Modeling results: 7th partition (a) and 15th partition (b)

Comparative study

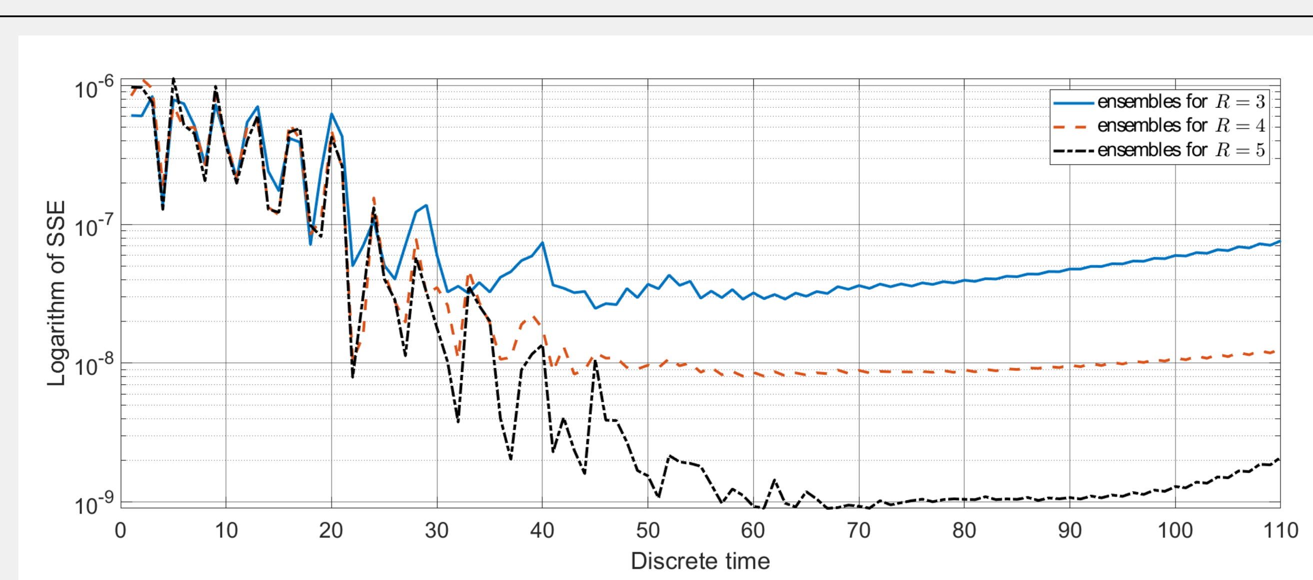

Figure 7. Modeling quality: blue–solid line – ensembles for $R=3$, red–dashed line – ensembles for $R=4$, black–dash-dot – ensembles for $R=5$

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