ELEC4410

Control Systems Design

Lecture 21: Design Considerations

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Outline

- Introduction
- Revision of Factors Limiting Closed Loop Bandwidth
- Dealing with Input Constraints in the Context of State Feedback and Observers.
- Trade-offs in State Feedback and Observers
- General Feedback Design Tips

Introduction

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Introduction

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- However, in feedback control system design there are a number of **limitations** that govern what is, and, what is not achievable: nonminimum phase zeros, unstable poles, time-delays, saturation, etc.
- These design limitations will impose constraints and tradeoffs in our choice of the desired closed-loop poles of the system.

Synthesis vs. Design

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> once we **know** where do we want the closed loop poles, they give us a way of obtaining a controller that will realise them.

- Thus, the true control system **design question** is:

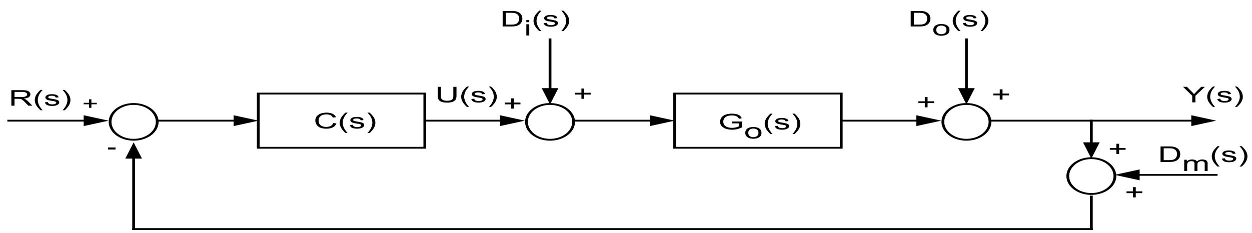
> Where should I assign the closed loop poles?

Factors Limiting Closed Loop Bandwidth

- An understanding of limitations is central to understanding control system design.
- Limitations arise due to:
  - Noise
  - Disturbances
  - Structural Limitations (nonminimum phase zeros, unstable poles and time delays)
  - Modelling Errors (uncertainty)
  - Actuator Limits (saturation)
- There will always exist trade-offs in closed loop performance subject to these limitations.

Factors Limiting Closed Loop Bandwidth: System Description

- Consider the SISO system in the figure below



- Ignoring the effect of initial conditions, the plant input and output can be expressed by the following equations:

$$U = \underbrace{\frac{C}{1+G_oC}}_{S_{uo}} R - \underbrace{\frac{C}{1+G_oC}}_{S_{uo}} D_m - \underbrace{\frac{C}{1+G_oC}}_{S_{uo}} D_o - \underbrace{\frac{G_oC}{1+G_oC}}_{T_o} D_i$$

$$Y = \underbrace{\frac{G_oC}{1+G_oC}}_{T_o} R - \underbrace{\frac{G_oC}{1+G_oC}}_{T_o} D_m + \underbrace{\frac{1}{1+G_oC}}_{S_o} D_o + \underbrace{\frac{G_o}{1+G_oC}}_{S_{io}} D_i$$

Factors Limiting Closed Loop Bandwidth: Noise

- Consider the effect of measurement noise on the plant output

$$\mathbf{Y}(s) = \mathbf{T}_0(s)\mathbf{D}_m(s)$$

- It can be easily seen that the effect of measurement noise can be attenuated if $|\mathbf{T}_0(j\omega)|$ is small in the region where $|\mathbf{D}_m(j\omega)|$ is significant.

> Given measurement noise is typically dominated by high frequencies, measurement noise will set an upper limit on the closed loop B.W.

Factors Limiting Closed Loop Bandwidth: Disturbances

- Consider the effect of disturbances on the plant output

$$\mathbf{Y}(s) = \mathbf{S}_0(s)\mathbf{D}_0(s) + \mathbf{S}_{i0}(s)\mathbf{D}_i(s)$$

- Assume that the input and output disturbances have significant energy only in the frequency bands, $\mathbf{B}_{wo}$ & $\mathbf{B}_{wi}$ respectively, thus it is clearly desirable to have small values of $|\mathbf{S}_0(j\omega)|$ & $|\mathbf{S}_{i0}(j\omega)|$ in the respective bands.
- Because $\mathbf{G}(s)$ is fixed this can only be achieved provided $\mathbf{S}_0(j\omega) \approx 0$ in the frequency band encompassing the union of $\mathbf{B}_{wo}$ and $\mathbf{B}_{wi}$.

> To achieve acceptable performance in the presence of disturbances will, in general, require a lower bound on the closed loop B.W.

Factors Limiting Closed Loop Bandwidth

- Whenever we make $T_0(s)$ small to satisfy measurement noise rejection, we necessarily increase $S_0(s)$ hence increasing sensitivity to output disturbances at that frequency.
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- The following closed loop properties cannot be addressed independently:
  - Sensitivity to measurement noise
  - Speed of disturbance rejection
- Tuning for one of these automatically affects the other.

Factors Limiting Closed Loop Bandwidth

- These trade-offs are made more precise by the following fundamental laws.
  - $Y(s) = -T_0(s)D_m(s)$

i.e. measurement noise, $d_m(t)$, is rejected only at frequencies where $\lvert T_0(j\omega)\rvert \approx 0$.

  - $S_0(s) = 1 - T_0(s)$

i.e. an output disturbance is rejected only at frequencies where $\lvert T_0(j\omega)\rvert \approx 1$.

Factors Limiting Closed Loop Bandwidth: Modelling Errors

- Another source of performance limitation is due to inadequate fidelity in the nominal model used as the basis of control system design.
- Modelling is normally good at low frequencies and deteriorates as the frequency increases, since then dynamic features neglected in the nominal model become significant.

Modelling error usually sets an upper bound on closed loop B.W.

Factors Limiting Closed Loop Bandwidth: Structural Limitations

- Structural limitations were discussed in depth in Lecture 8.
- If the magnitude of the real part of the dominant closed loop poles is greater than the smallest R.H.P. zero, then large undershoot is inevitable.

> The closed loop B.W. should in practise be set less than the smallest N.M.P zero.

- If the magnitude of the real part of the dominant closed loop poles is less than the magnitude of the largest unstable open-loop pole, then significant overshoot will occur.

> The closed loop B.W. should be set greater than the real part of any unstable pole.

Factors Limiting Closed Loop Bandwidth: Structural Limitations

- If the magnitude of the real part of the dominant closed loop poles is greater than the magnitude of the smallest stable open loop zero then significant overshoot will occur.
  - One idea is to cancel the zero in the closed loop by placing them in the denominator of the controller. However, they will then appear in the numerator of the input sensitivity, which may be okay as input disturbances can be significantly attenuated by passage through the plant.
  - Alternatively, use a 2 D.O.F. controller and cancel them in the set point response only.

Factors Limiting Closed Loop Bandwidth: Structural Limitations

- One of the most common sources of structural limitation in process control applications is due to process delays.
- Delays limit disturbance rejection by requiring that a delay occur before the disturbance can be cancelled.
- Delays also limit the bandwidth due to the impact of model errors.

Process delays set an upper bound on closed loop B.W.

Actuator Saturation

Actuators are a source of performance limitations in control systems. They impose

- **Amplitude limits** by constraining the amplitude of the control signal,
- **Slew rate limits** by limiting the rate of change of the control signal.

Actuator Saturation: Amplitude Limits

In a 1 DOF control loop the controller output is given by

$$U(s) = S_{u0}(s)\,(R(s) - D_0(s))$$

- Peaks in the control action usually occur as a result of large fast changes in either the reference $r(t)$ or the output disturbance $d_0(t)$. Input disturbances, $d_i(t)$, are usually attenuated by the plant and hence are neglected here.

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- Peaks in the control action usually occur as a result of large fast changes in either the reference $r(t)$ or the output disturbance $d_0(t)$. Input disturbances, $d_i(t)$, are usually attenuated by the plant and hence are neglected here.
- If the closed loop B.W. is greater than the open loop dynamics of $G_0(s)$ then the controller sensitivity, $S_{u0}(s)$ will significantly enhance the high frequency components in $R(s)$ and $D(s)$. This is easily seen as the control sensitivity is defined as,

$$S_{u0}(s) \triangleq \frac{T_0(s)}{G_0(s)}$$

Actuator Saturation: Slew Rate Limits

On the other hand, actuator saturation limits the maximum speed at which the actuator can change position.

- The rate of change of the input can be expressed as:

$$\begin{aligned} sU(s) &= S_{u0}(s)\,(sR(s) - sD_0(s)) \\ &= \frac{T_0(s)}{G_0(s)}\,(sR(s) - sD_0(s)) \end{aligned}$$

- If the closed loop B.W. is much larger than that of the plant dynamics then the rate of change of the input signal will be large for fast changes in $r(t)$ and $d_0(t)$.

Actuator Saturation

In conclusion:

> To avoid amplitude and slew rate limit problems, it will be necessary to place an upper limit on the closed loop B.W.

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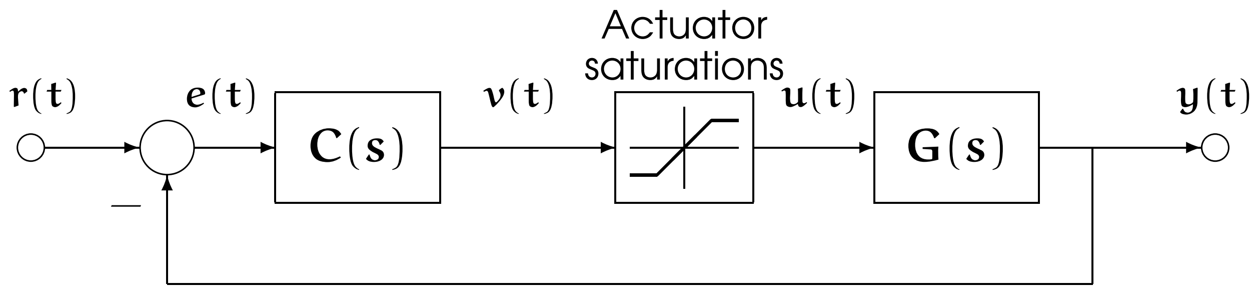
Actuator Saturation: Integrator Windup

Amplitude limits in conjunction with a controller with integral action induces integrator windup, which deteriorates closed-loop performance.

- Note that when the actuator hits its limits, the loop is virtually **open**!



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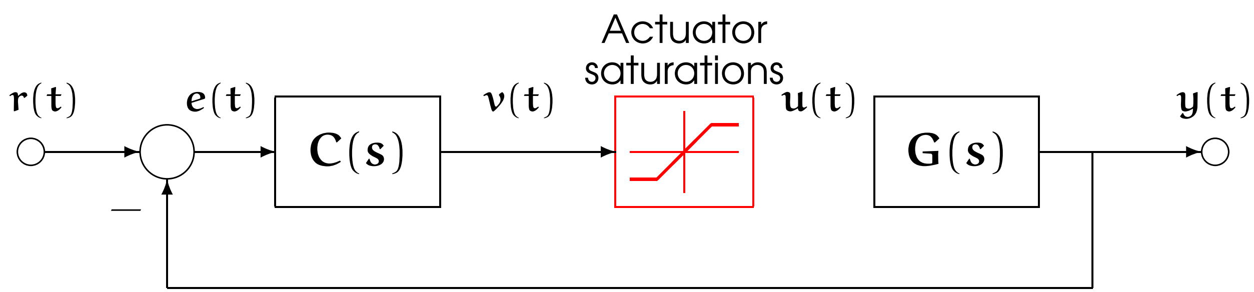

- Indeed, when the error $e(t)$ is so large that a command $v(t)$ to the actuator **exceeds** the saturation levels, the command cannot be realised and the control loop is broken.

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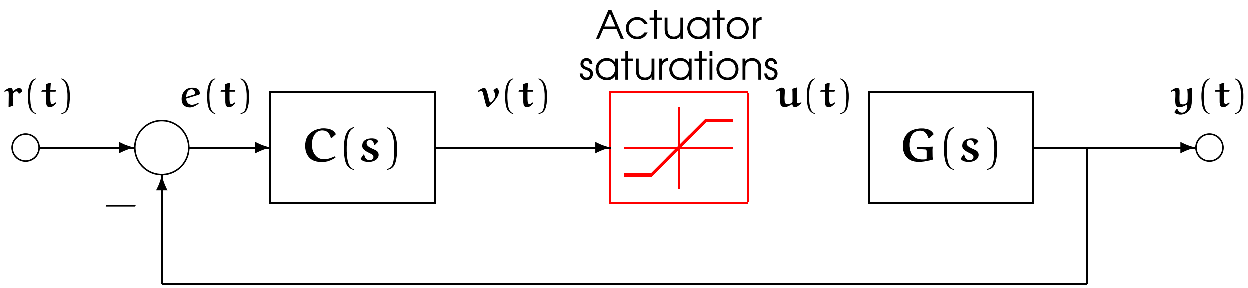


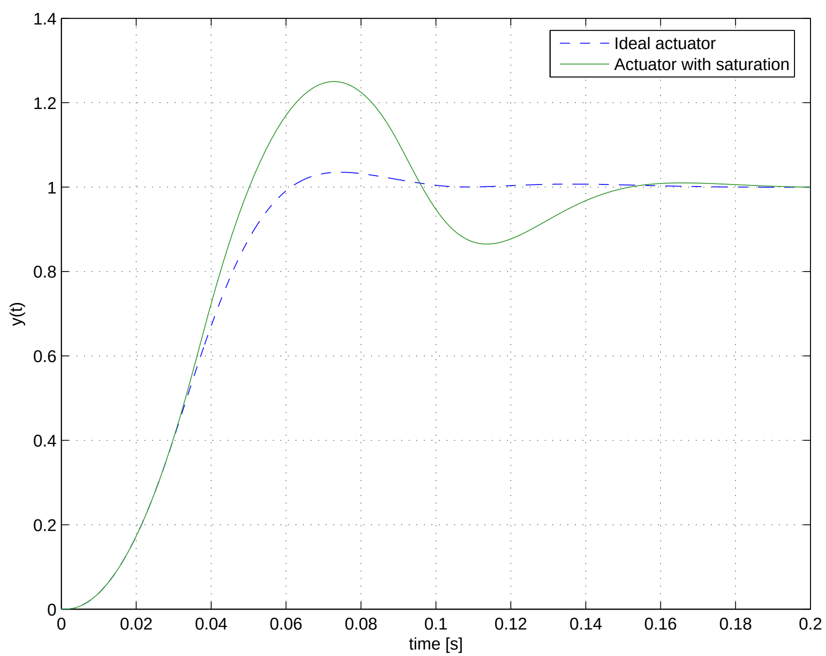
- Indeed, when the error $e(t)$ is so large that a command $v(t)$ to the actuator **exceeds** the saturation levels, the command cannot be realised and the control loop is broken.

If the controller contains an integrator, the broken loop becomes unstable, so that $v(t)$ will continue to grow until $v(t) \gg u_{\mathrm{sat}}$.

Actuator Saturation: Integrator Windup

Suppose now that the error signal $e(t)$ eventually becomes small, so that the command $v(t)$ can be accommodated between the saturation limits of the actuator.

It may take a long time for the integrator to "come off saturation", which will result in a very sluggish response, deteriorating the system closed-loop performance, or even inducing instability.



Antiwindup Schemes

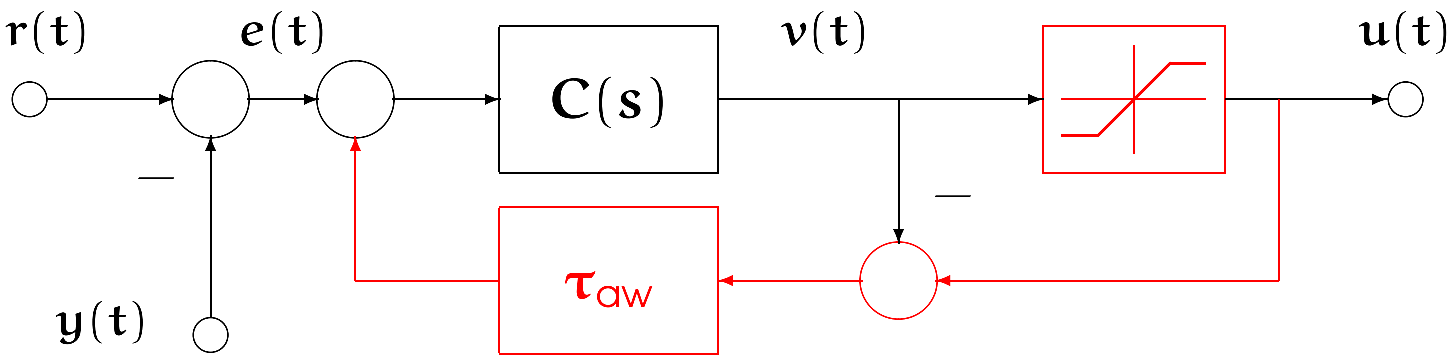
We have seen in IMC design some structures to implement in the controller so that we can compensate the effects of integrator windup.

Recall the **principles of antiwindup compensation:**

- The states of the controller should be driven by the actual input to the plant.
- The states of the controller should have a stable realisation when the actuator saturates.

Antiwindup Schemes

A **general antiwindup compensation** scheme is the following:



The parameter τ_{aw} can be used to tune the response.

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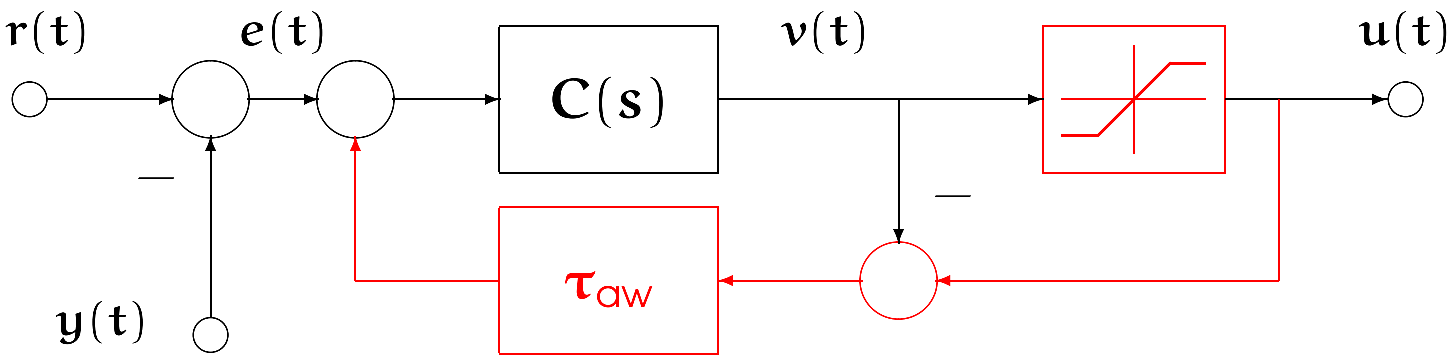

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The saturation block is a **model of the actuator**. If $C(s)$ contains an integration (as in PIDs), this scheme prevents instability of the controller when the actuator saturates.

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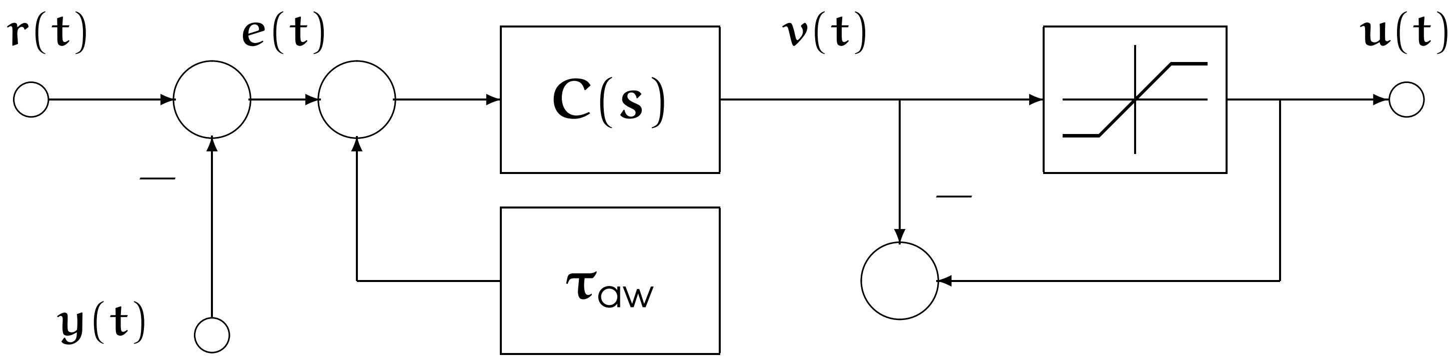


The parameter τ_{aw} can be used to tune the response.

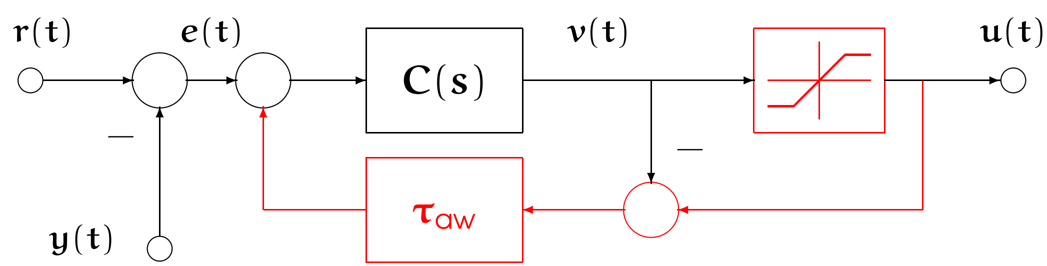
The saturation block is a **model of the actuator**. If $C(s)$ contains an integration (as in PIDs), this scheme prevents instability of the controller when the actuator saturates.

On the other hand, when there is no saturation, the τ_{aw} compensation loop does not act.

Antiwindup in State Feedback

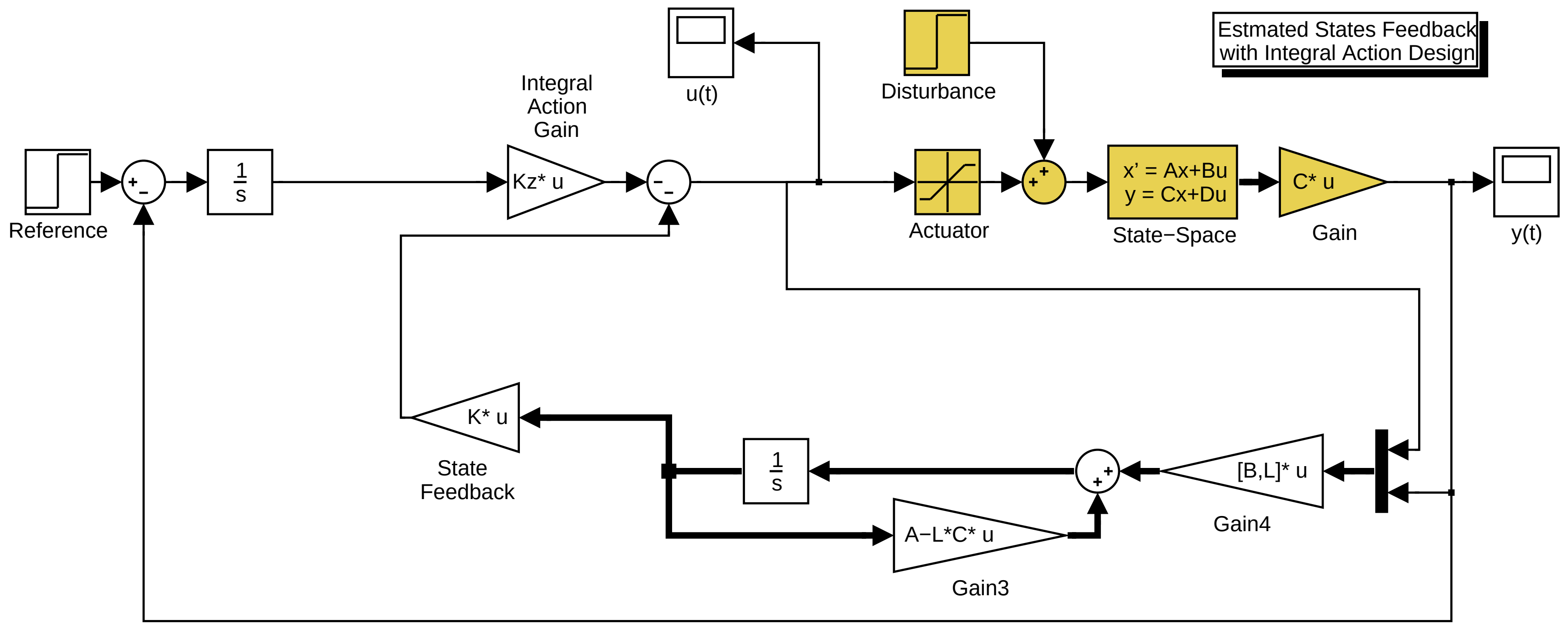
Returning to state space controllers:

- We have seen how to implement a state controller with feedback from estimated states and integral action for robust tracking.
- We show how we can modify the implementation of this controller to incorporate **antiwindup**.
- To do so, we will make some block diagram transformations to bring the integrator next to the actuator in order to implement the general antiwindup compensation.



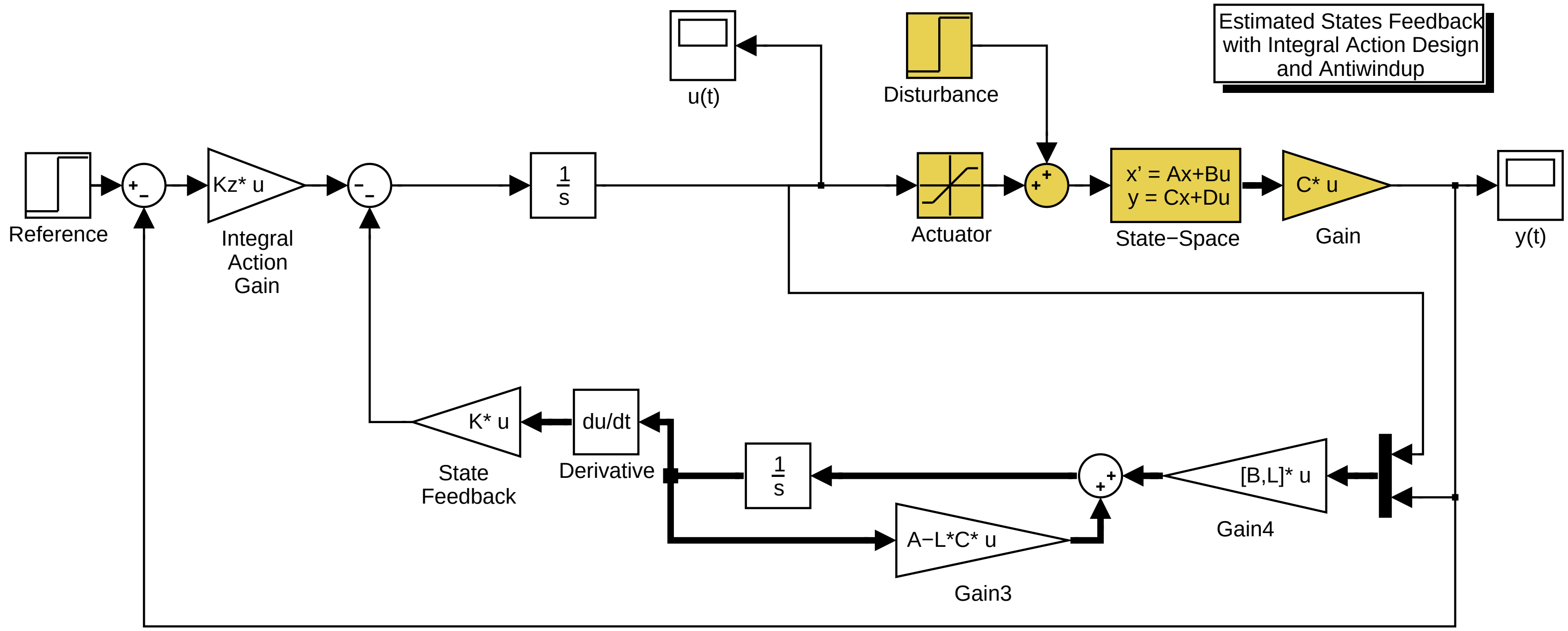
Antiwindup in State Feedback

- Start from the original closed loop configuration:



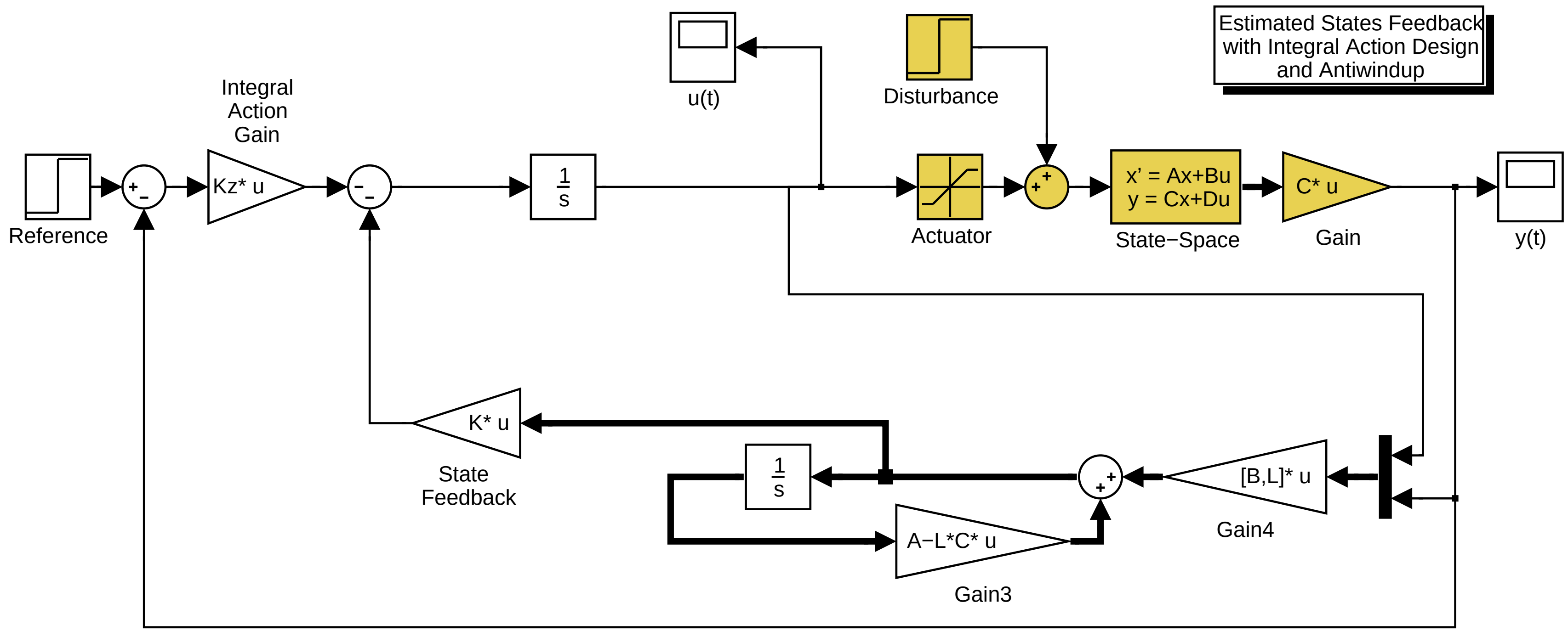
Antiwindup in State Feedback

- Interchange state feedback loop and integrator



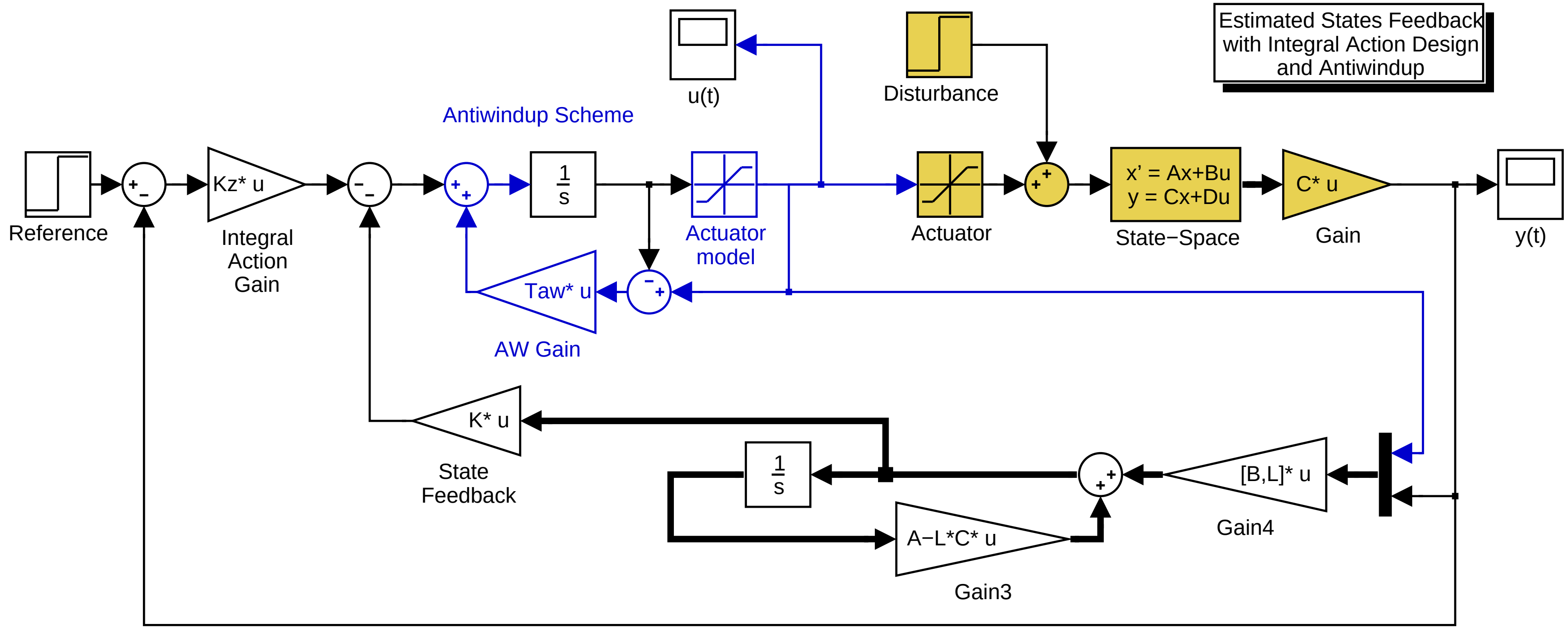
Antiwindup in State Feedback

- Reimplement state feedback loop to eliminate $\mathbf{d}/\mathbf{dt}$:



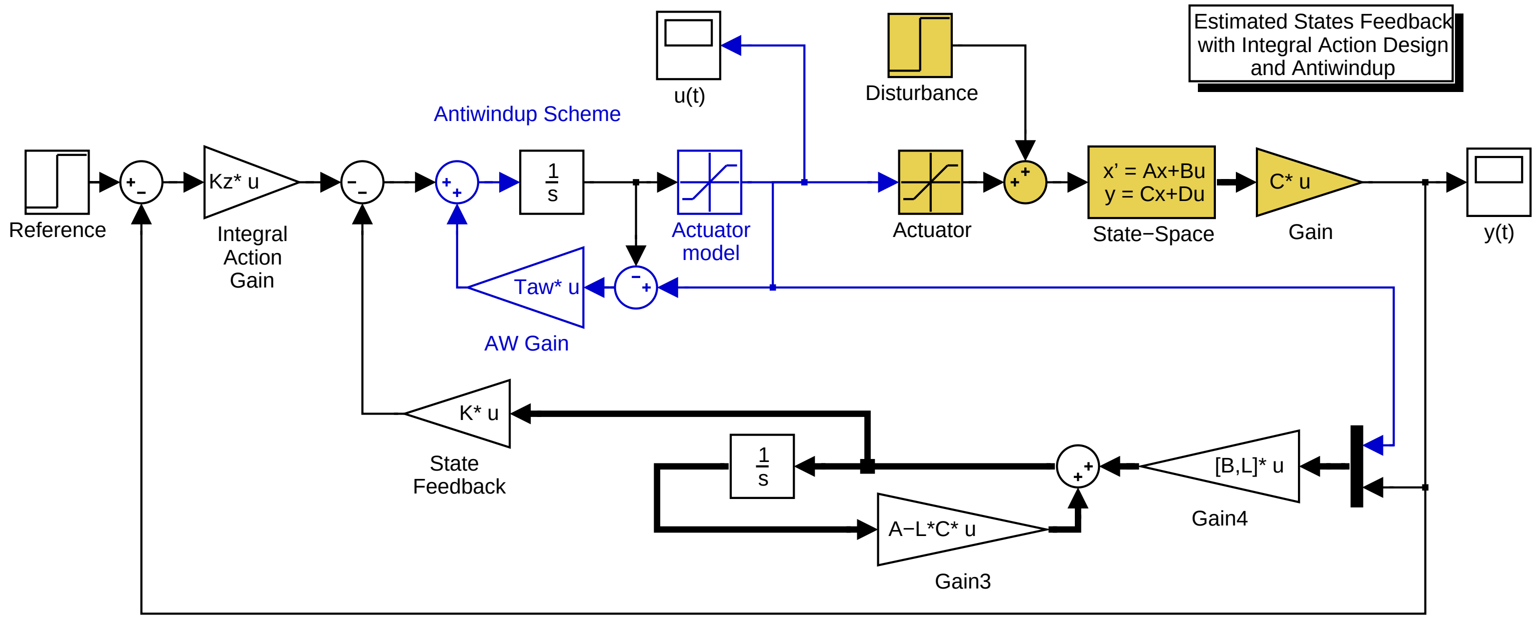
Antiwindup in State Feedback

- Incorporate antiwindup compensation to the integrator:



This scheme will compensate integrator windup **only** when needed.

Antiwindup in State Feedback



Note that the **principles of antiwindup compensation are satisfied**:

- The states of the controller are driven by the actual input to the plant (observer and integral action).
- The states of the controller have a stable realisation when the actuator saturates (observer and integral action).

Antiwindup in State Feedback

Example (HDD output feedback controller).

Consider the position control
of a Hard Disk Drive given by the model

$$G(s) = \frac{K_0 \omega_r^2}{s^2(s^2 + 2\xi\omega_r s + \omega_r^2)}$$

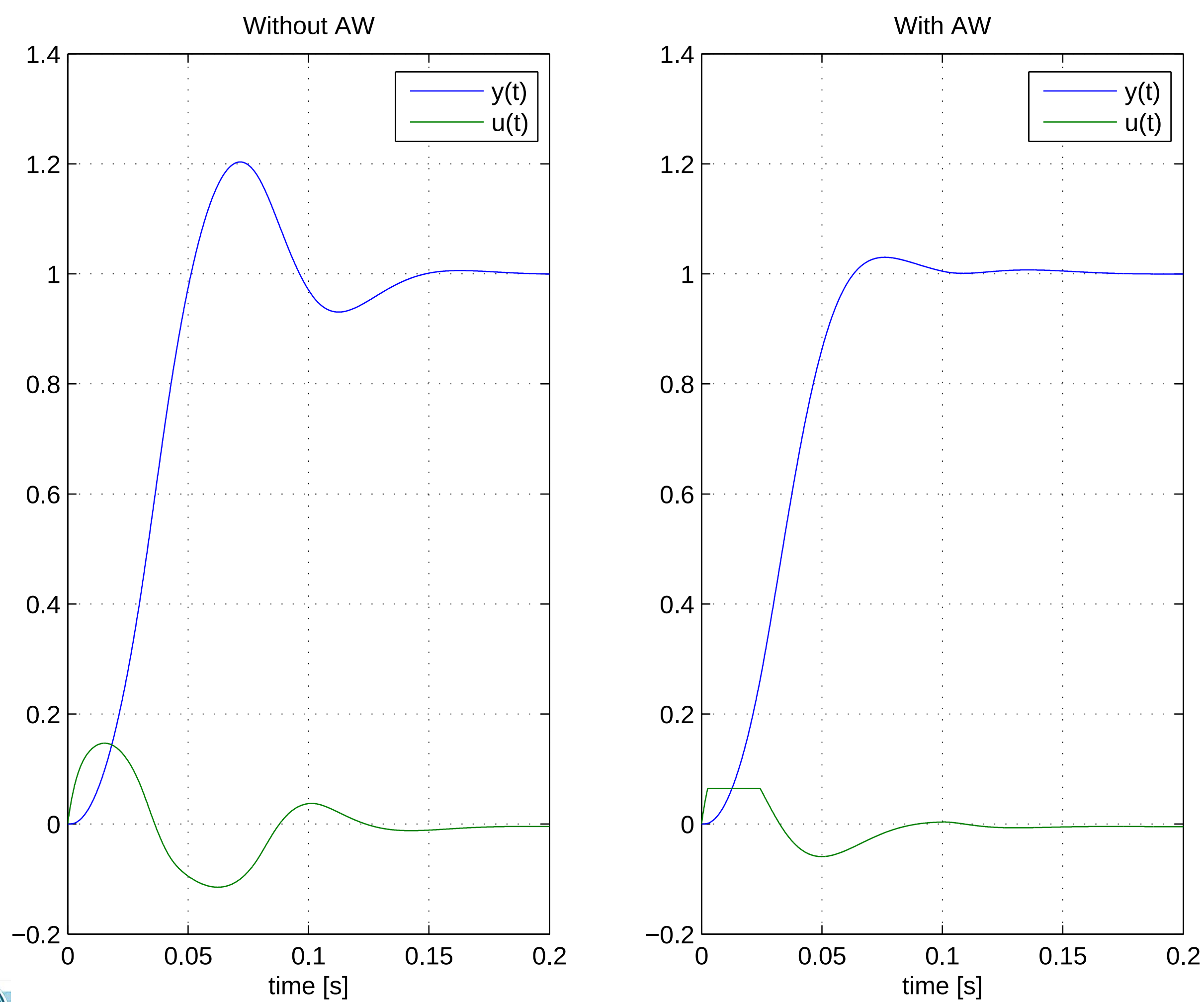
Ignoring any input saturation, we designed an observer-based state feedback controller with integral action to achieve

$$\boxed{\text{max overshoot} < 3\%, \quad \text{settling time} < 0.1\text{s}.}$$

We compare the closed loop performance of this system without and with antiwindup compensation for an actuator saturation level of $u_{\text{sat}} = 0.065$, which corresponds approximately to 100% saturation for the maximum peak command required.

Antiwindup in State Feedback

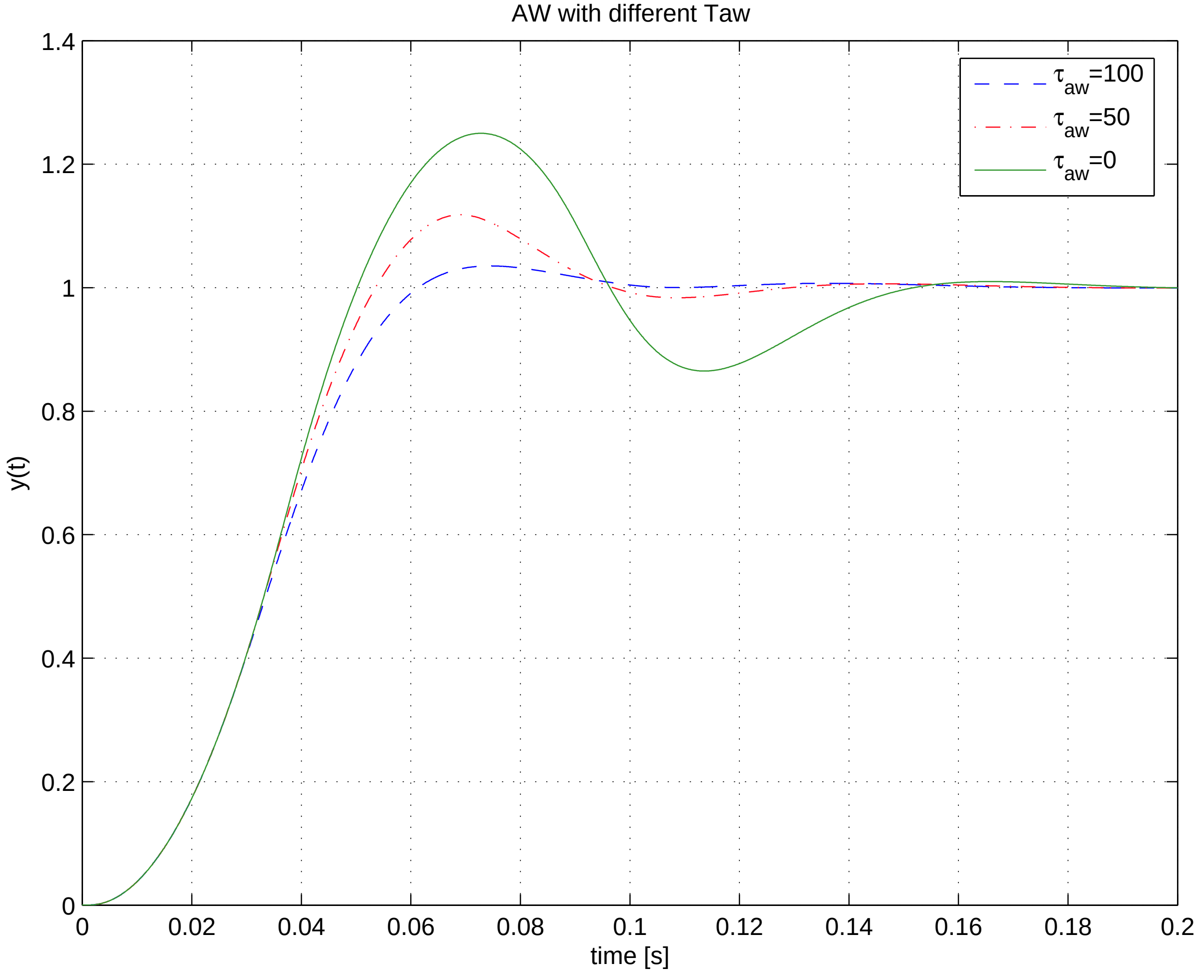
Example (HDD output feedback controller). We can see how performance deteriorates with actuator saturation — overshoot jumps from 3% to 20%, and settling time from 0.1 to 0.15 s.

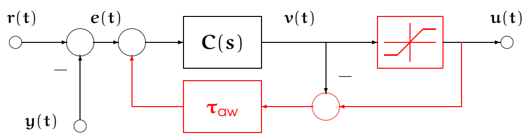


On the other hand, the ideal performance is almost completely recovered with the antiwindup compensation.

Antiwindup in State Feedback

Example (HDD output feedback controller). This plot shows the effect of the parameter τ_{aw} on the compensated closed-loop response ($\tau_{aw} = 0 \Rightarrow$ no compensation).





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Trade-offs in State Feedback & Observers

- Under the assumption of controllability and by a suitable choice of the feedback gain $\mathbf{K}$, the closed loop poles could be assigned to any desired set of locations.

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- However, if the closed loop modes are chosen much faster than those of the plant, then the gain $\mathbf{K}$ will be large, leading to a large plant input $\mathbf{u}(\mathbf{t})$.

Trade-offs in State Feedback & Observers

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- However, if the closed loop modes are chosen much faster than those of the plant, then the gain $\mathbf{K}$ will be large, leading to a large plant input $\mathbf{u}(t)$.
- A similar problem arises in state estimation. Consider the state space model:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{5}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{v}(t) \tag{6}$$

where $\mathbf{v}(t)$ is the measurement noise.

Trade-offs in State Feedback & Observers

Then the state estimate and the estimation error are:

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}\hat{\mathbf{x}}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{L}\mathbf{C}(\mathbf{x}(t) - \hat{\mathbf{x}}(t)) + \mathbf{L}\boldsymbol{v}(t) \qquad (7)$$

$$\dot{\tilde{\mathbf{x}}}(t) = (\mathbf{A} - \mathbf{L}\mathbf{C})\tilde{\mathbf{x}}(t) - \mathbf{L}\boldsymbol{v}(t) \qquad (8)$$

Applying the Laplace transform to (4),

$$\tilde{\mathbf{X}}(s) = [s\mathbf{I} - \mathbf{A} + \mathbf{L}\mathbf{C}]^{-1}\tilde{\mathbf{x}}(0) - [s\mathbf{I} - \mathbf{A} + \mathbf{L}\mathbf{C}]^{-1}\mathbf{L}\mathbf{V}(s) \qquad (9)$$

- Here we see that if $\mathbf{L}$ is chosen to place the eigenvalues of $\mathbf{A} - \mathbf{L}\mathbf{C}$ well into the left half plane, we will quickly eliminate the effect of the initial error $\tilde{\mathbf{x}}(0)$.

Trade-offs in State Feedback & Observers

- However, this will almost certainly require a large value for $\mathbf{L}$. We then see that the second term on the right hand side of equation (5) will enhance the effect of the measurement noise, since this is usually a high frequency signal.

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- How to select a desired set of eigenvalues? Of course this depends on many performance criteria (rise time, settling time, overshoot, etc..)

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- We then need to compromise between speed of response and noise immunity.
- How to select a desired set of eigenvalues? Of course this depends on many performance criteria (rise time, settling time, overshoot, etc..)
- We must remember that the response of a system not only depends on the position of the poles but also on the zeros and, of course, the limits placed on the control signal by the actuator.

Trade-offs in State Feedback & Observers

As a guide we may place all the eigenvalues inside the region denoted by $\mathbf{C}$ in the figure below.

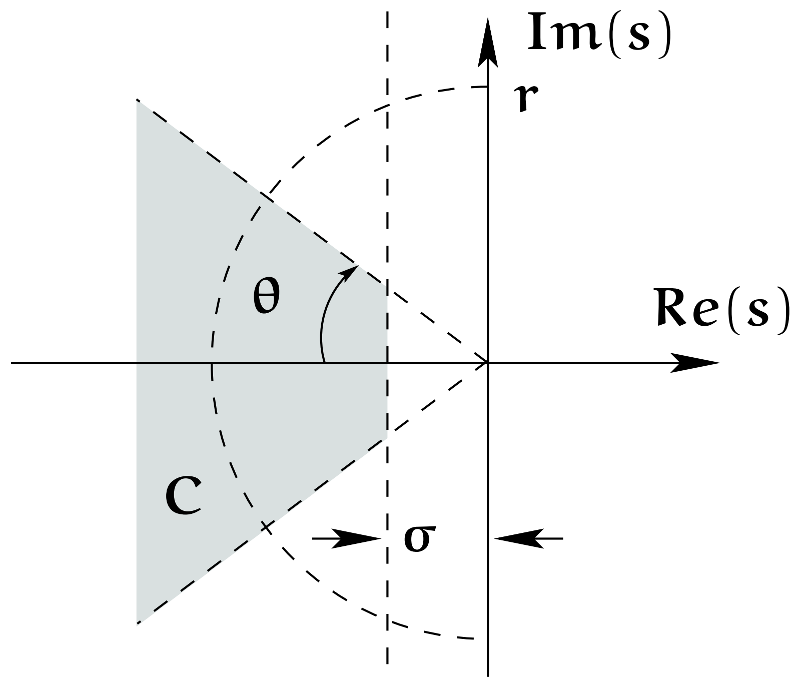

Figure 1:

Trade-offs in State Feedback & Observers

Several points to note about this region and the expected performance are:

- The region is bounded by a vertical line. The greater the distance this line is from the imaginary axis, the faster the response.
- The region is also bounded by 2 straight lines emanating from the origin with angle θ. The larger the angle, the larger the overshoot.
- If all eigenvalues are placed at one point or grouped in a very small region, then usually the response will be slow and the actuating signal large.

Trade-offs in State Feedback & Observers

- It is better to place eigenvalues around a circle with radius $\mathbf{r}$ inside the sector as shown. The larger the radius,
  - the faster the response and of course the larger the control signal.
  - the larger the closed loop B.W. resulting in a system more susceptible to noise.

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General Feedback Design Tips

How to choose an appropriate Closed Loop Bandwidth? A general recipe you may wish to consider is:

- The B.W. must be large relative to the location of unstable poles. In particular, a suggestion is:

$$\omega_b \geq 5 \sum_{i \in U} p_i \tag{10}$$

  where U denotes the set of unstable poles p_i.

- The B.W. must be small relative to the location of R.H.P. zeros ξ_i and to time delay τ. In particular, a suggestion is:

$$\omega_b \leq \left(\frac{\tau}{2} + \sum_{i \in U'} \frac{1}{\xi_i} \right)^{-1} \tag{11}$$

  where U' denotes the set of R.H.P. zeros ξ_i.

General Feedback Design Tips

- The B.W. must be small relative to the frequency where the relative modelling error $\left|\frac{\mathbf{G}_\Delta}{\mathbf{G}_0}\right|$ approaches 1.
- The B.W. is constrained by the input amplitude and slew rate limits.

Having chosen the B.W. the following are additional guidelines.

- Any stable, well damped zeros within the B.W. should be cancelled in the controller.
- Any stable, well damped poles within the B.W. may be cancelled provided they are not close to the origin compared to the B.W.

General Feedback Design Tips

- Let k denote the number of open loop poles in the plant and controller that are close to the origin relative to the B.W. (in this sense, the distance to the origin be $< \frac{1}{5}\omega_b$); then there should be $k - 1$ closed loop poles at stable well damped locations whose distance to the origin is about $\frac{1}{5}\omega_b$.
- The poles not constrained by the last 3 guidelines should be placed at stable well damped locations as follows:
  - Not more than 2 at ω_b.
  - The remainder at greater than $3\omega_b$
- If any of the controller poles or zeros are unstable then alternative measurements should be used.

General Feedback Design: Further Ideas

Dominant 2nd Order Poles

For a second order system, rise time, settling time and overshoot can be directly deduced from the pole locations. A simple technique to utilise this knowledge to give performance measures in a higher order system is:

- Choose the closed loop poles for a higher order system as a desired pair of dominant 2nd order poles.
- Select the rest of the poles to have real parts corresponding to sufficiently damped modes so that the system will mimic a 2nd order response with reasonable control effort.
- Ensure that the zeros are far enough into the L.H.P to avoid having any appreciable effect on the 2nd order behaviour.

General Feedback Design: Further Ideas

Observer Pole Selection

- Can use the same ideas as for the feedback design.
- As a rule of thumb observer poles should be chosen to be a factor of $(2 \text{ to } 6)\times$ faster than the controller poles. This ensures the observer errors decay faster than the desired closed loop dynamics allowing the controller poles to dominate the total response.
- If sensor noise is a problem then the observer poles may be chosen slower than $2\times$ the controller poles. This would yield a system with lower bandwidth and more noise smoothing.
- Unlike the controller, the observer output is a number in a computer. Depending on the computer numerical precision, there are no real limits on the size of this number. However, **measurement noise will limit the speed of the observer**.

Summary

- Revision of Factors that Limit Closed Loop Bandwidth
  - Noise
  - Disturbances
  - Modelling Errors
  - Structural Limitations
  - Actuator Limits
- Input Constraints in State Feedback and Observers
- Trade-offs in State Feedback and Observers
- General Control System Design Tips