FEL3210 Multivariable Feedback Control

Elling W. Jacobsen

Automatic Control Lab, KTH

jacobsen@kth.se

http://www.ee.kth.se/~jacobsen/multi_12

Lecture 1: Introduction, classical SISO feedback control

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The Practical

- 8 lectures
  - Tuesdays 13-15
  - slides on homepage
  - reading assignments on homepage
- Course literature
  - *Skogestad and Postlethwaite, Multivariable Feedback Control, 2nd ed.*
  - *Supporting text: Zhou, Doyle and Glover, Robust and Optimal Control*
- 8 homeworks, compulsory
  - download from homepage after each lecture, hand in at next lecture
  - require Matlab with Robust Control toolbox
- 1-day take home open book exam, within 2 weeks after last lecture

Course Content

Feedback control of systems described by LTI models with a focus on applicability to real problems

- *frequency domain* analysis and design;
  - extension of classical SISO methods to MIMO systems
  - optimization in frequency domain
- *system controllability*; what can be achieved with feedback in a given system?

- model uncertainty: *robust stability* and *robust performance*

Course goals

After completed course you should be able to

- quantify the performance that can be achieved with feedback for a given system
- analyze feedback systems with respect to stability and performance in the presence of structured and unstructured model uncertainty
- design/synthesize controllers for robust performance

Lecture Plan

L1: Introduction, classical SISO feedback control (Ch.1-2)

L2: Performance limitations in SISO feedback (Ch. 5)

L3: Introduction to MIMO systems, excerpts from Linear Systems Theory (Ch. 3-4)

L4: Performance limitations in MIMO feedback (Ch. 6)

L5: Uncertainty and robust stability (Ch. 7-8)

L6: Robust performance (Ch. 7-8)

L7: Controller synthesis and design (Ch. 9-10)

L8: LMI formulations, control structure design, summary (Ch. 10, 12)

The Control Problem

Control problems usually formulated in terms of signal tracking

$$y = Gu + G_d d$$

y - output / controlled variable

u - input / manipulated variable

d - disturbance

r - reference, setpoint

- *Regulator problem:* attenuate effect of d on y
- *Servo problem:* make y follow r

Control objective: make control error $e = r - y$ "small" in the presence of d, by adding feedback $u = \mathcal{C}(y, r)$

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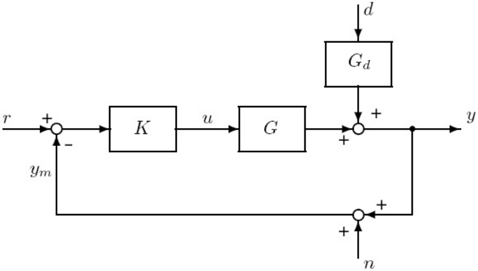
Why Feedback?



Why not $u = G^{-1}r - G^{-1}G_d d$?

1. model uncertainty - uncertain knowledge of system behavior
2. signal uncertainty - unmeasured disturbances
3. instability

Cost of feedback?

- potentially induce instability
- introduce measurement noise into process

Fact 1: Feedback has its limitations

- feedback is a simple and potentially extremely powerful tool, but with limitations to what can be achieved
- control performance depends on controller **AND** system

Ziegler and Nichols (1943): *In the application of automatic controllers, it is important to realize that controller and process form a unit; credit or discredit for results obtained are attributable to one as much as the other. . . . The finest controller made, when applied to a miserably designed process, may not deliver the desired performance. True, on badly designed processes, advanced controllers are able to eke out better results than older models, but on these processes, there is a definite end point which can be approached by instrumentation and it falls short of perfection.*

There is always a hard limit for what can be achieved with feedback!

Approaches to Control Design

- "Traditional":
  1. specify desired performance
  2. design controller that meets specifications
  3. if 2. fails, try more advanced controller and repeat from 2 (or go back to 1).
- This course:
  1. determine achievable performance
  2. if specifications not feasible, change specifications or the system
  3. design controller using your favorite method

Fact 2: Models are always uncertain

Models (G, G_d) always inaccurate, e.g., true system

$$G_p = G + E$$

with $E =$ "uncertainty", or "perturbation" (unknown)

Definitions for closed loop:

- **Nominal stability (NS):** system stable with no model uncertainty
- **Nominal performance (NP):** systems satisifies performance requirements with no model uncertainty
- **Robust stability (RS):** system stable for "all" perturbations E
- **Robust performance (RP):** system satisifies performance requirements for "all" perturbations E

System Representations

- State-space representation

$$\dot{x} = Ax(t) + Bu(t), \quad x \in \mathbf{R}^n, u \in \mathbf{R}^p$$
$$y(t) = Cx(t) + Du(t), \quad y \in \mathbf{R}^l$$

- Transfer-function

$$Y(s) = G(s)U(s) \; ; \quad G(s) = C(sI - A)^{-1}B + D$$

- Frequency response

$$Y(j\omega) = G(j\omega)U(j\omega)$$

Sometimes we write

$$\begin{bmatrix} \dot{x} \\ y \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \; ; \quad G = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$$

A Brief History of Control

- Classical, 30's-50's: frequency domain methods
  Bode, Nyquist, Nichols, . . .
  + yields insight (loop shaping)
  + address uncertainty (gain and phase margins)

  ÷ only applicable to SISO systems

- Modern, 60's-70's: state-space optimal control
  Bellman, Pontryagin, Kalman, . . .
  + control cast as optimization problem
  + applicable to MIMO systems (LQG)

  ÷ do not address model uncertainty.
  ÷ LQG has no guaranteed stability margins
  ÷ no clear link to classical methods

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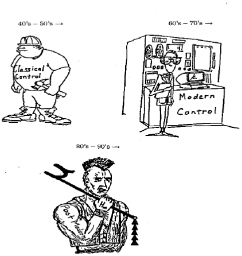
A Brief History of Control

- Postmodern, 80's-90's: robust control
  Zames, Francis, Doyle, . . .
  - $+$ frequency domain methods for MIMO systems
  - $+$ explicitly adress model uncertainty
  - $+$ control cast as optimization problem ($\mathcal{H}_2, \mathcal{H}_\infty$)
  - $+$ links classical and modern approaches; *"formulate and analyze in input-output domain, compute in state-space"*
  - $\div$ high controller orders, some computational issues, ...



Next: repetition of classic SISO control

Rational Transfer Functions

$$G(s) = \frac{\beta_{n_z} s^{n_z} + \ldots + \beta_1 s + \beta_0}{s^n + \alpha_{n-1} s^{n-1} + \ldots + \alpha_1 s + \alpha_0}$$

n - order of denominator = system order

n_z - order of numerator

$n - n_z$ - pole excess, or relative order

Definitions

- A system is **strictly proper** if $\lim_{s \to \infty} G(s) = 0$ $(n - n_z > 0)$
- A system is **semi-proper** if $\lim_{s \to \infty} G(s) = D \neq 0$ $(n - n_z = 0)$
- A system is **improper** if $G(s) \to \infty$ as $s \to \infty$ $(n - n_z < 0)$

All physical systems are strictly proper, certain closed-loop transfer-functions are semi-proper.

Irrational transfer-functions:

$$G(s) = \frac{\beta_{n_z} s^{n_z} + \ldots + \beta_1 s + \beta_0}{s^n + \alpha_{n-1} s^{n-1} + \ldots + \alpha_1 s + \alpha_0} e^{\theta s}$$

- Defintions of properness as above.
- Causal if $\theta \leq 0$, non-causal if $\theta > 0$ (output depends on future inputs)

Scaling - simplifies analysis and design

Unscaled model:

$$\hat{y} = \hat{G}\hat{u} + \hat{G}_d\hat{d} \; ; \quad \hat{e} = \hat{r} - \hat{y}$$

Scale all variables to have expected/allowed magnitude less than 1:

$$u = \frac{\hat{u}}{\hat{u}_{max}} \; ; \quad d = \frac{\hat{d}}{\hat{d}_{max}} \; ; \quad y = \frac{\hat{y}}{\hat{e}_{max}} \; ; \quad e = \frac{\hat{e}}{\hat{e}_{max}} \; ; \quad r = \frac{\hat{r}}{\hat{e}_{max}}$$

Introduce $D_d = \hat{d}_{max} \; ; \quad D_u = \hat{u}_{max} \; ; \quad D_e = \hat{e}_{max}$

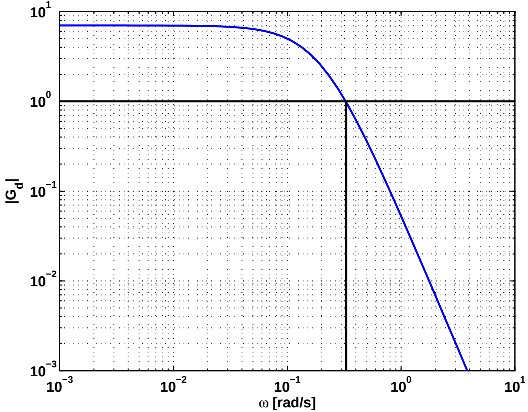
Scaled model:

$$y = \underbrace{D_e^{-1}\hat{G}D_u}_{G}\, u + \underbrace{D_e^{-1}\hat{G}_d D_d}_{G_d}\, d$$

- In the scaled model all signals should have magnitude less than 1, i.e., expected $|d| < 1$ and acceptable $|e| < 1$

Example: scaled frequency response

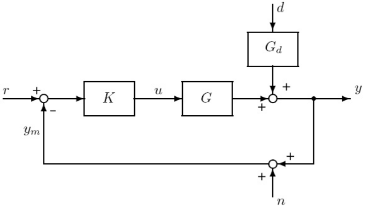
Bode plot for $|G_d(j\omega)|$:



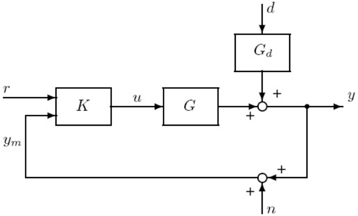
Need disturbance attenuation for frequencies where $|G_d(j\omega)| > 1$, i.e., for $\omega < 0.33$ *rad*/*s*

Control Structures

- 1-Degree of freedom

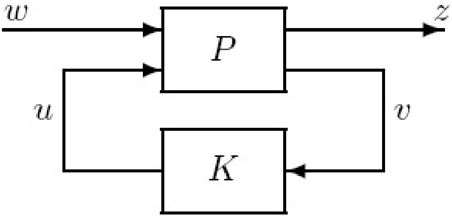


- 2-Degrees of freedom



Control Structures

- General control structure

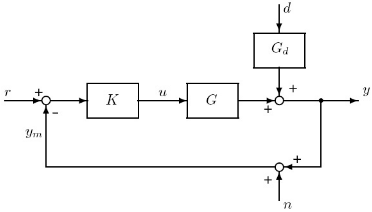


P - generalized system, *K* - controller

w - exogeneous inputs (d, r, n)	*z* - exogeneous outputs (e, u)
u - manipulated inputs	*v* - measurements, setpoints

Objective: minimize gain from *w* to *z*. With appropriate weights/scaling, make gain smaller than 1

Closed-Loop Transfer Functions - 1-DOF structure



Closed-loop transfer-functions

$$y = \underbrace{(I + GK)^{-1} GK}_{T} \, r + \underbrace{(I + GK)^{-1}}_{S} \, G_d d - \underbrace{(I + GK)^{-1} GK}_{T} \, n$$

control error

$$e = r - y = -Sr + SG_d d - Tn$$

input

$$u = KSr - KSG_d d - KSn$$

Important transfer functions

Introduce the loop gain $L = GK$

$$S = (I + L)^{-1} \; ; \quad T = (I + L)^{-1} L$$

$$\Downarrow$$

$$S + T = 1$$

S - the sensitivity function

T - the complimentary sensitivity function

The name ”Sensitivity”

- Bode: sensitivity of T to relative model errors

$$S = (dT/T)/(dG/G)$$

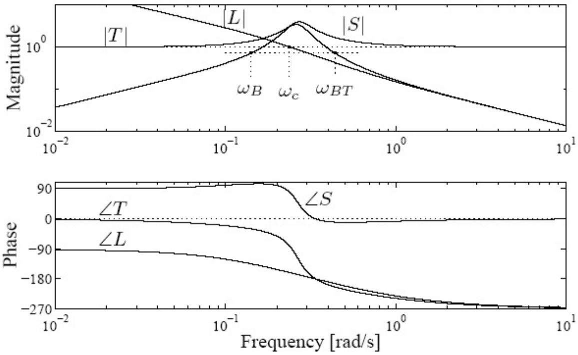
- But, also effect of feedback on sensitivity to disturbances

$$y = SG_d d$$

$|S(j\omega)| < 1$: reduce disturbance sensitivity

$|S(j\omega)| > 1$: increase disturbance sensitivity

Frequency Plots



Definitions:

- crossover frequency ω_c: $|L(j\omega_c)| = 1$
- bandwidth ω_B: $|S(j\omega_B)| = 1/\sqrt{2}$
- bandwidth for T, ω_{BT}: $|T(j\omega_{BT})| = 1/\sqrt{2}$

Bandwidth and Crossover Frequency

- Effective feedback for frequencies where $|S(j\omega)| < 1$, i.e., up to bandwidth ω_B

$$|S(j\omega)| < 1, \ \omega \in [0, \omega_B]$$

- Bandwidth and crossover frequencies:

$$\omega_B < \omega_c < \omega_{BT}$$

  Proof: see (2.53) in book

- Typically assume

$$\omega_c \approx \omega_B$$

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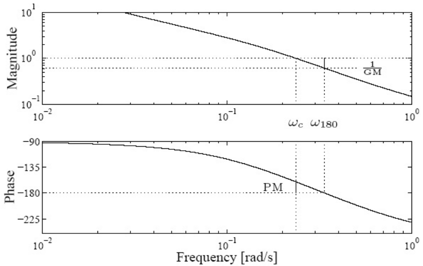
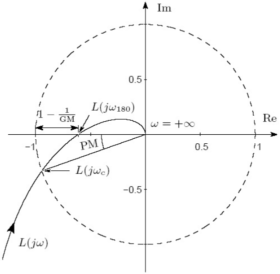
Stability margins

Gain margin GM and phase margin PM - robustness measures

Bode diagram



Nyquist diagram



Sensitivity peaks

Stability margins and performance are related

- $|S|^{-1} = |1 + L(j\omega)|$ is distance from $L(j\omega)$ to critical point -1 in Nyquist diagram
- define $M_S = \max_\omega |S(j\omega)|$; $\quad M_T = \max_\omega |T(j\omega)|$, then

$$M_S \geq \frac{1}{PM} \qquad M_S \geq \frac{GM}{GM-1}$$

$$M_T \geq \frac{1}{PM} \qquad M_T \geq \frac{1}{GM-1}$$

  – obtained by considering the loop gain at L at ω_c and ω_{180}, respectively

Controller Design

Three main approaches:

1. **Shaping transfer-functions**
   a. **Loop shaping (classic):** use controller K to shape loop gain $L(j\omega)$
   b. **Shaping the closed loop:** shape S, T etc, using optimization based methods

2. **Signal based approaches:** minimize signals, i.e., control error e and input u, given characteristics of inputs d, r, n.

3. **Numerical optimization:** optimize "real" control objectives, e.g., rise time and overshoot for step responses.

Classic Loop Shaping - shaping $|L|$

recall

$$e = -\underbrace{\frac{1}{1+L}}_{S} r + \underbrace{\frac{1}{I+L}}_{S} d - \underbrace{\frac{L}{I+L}}_{T} n$$

Fundamental trade-offs:

- setpoint following: $|L|$ large
- disturbance attenuation: $|L|$ large
- noise propagation: $|L|$ small

Also:

$$u = KSr - KSG_d d - KSn$$

- input usage: $|K|$ small

Resolving the trade-off

Typically: make $|L|$ large in frequency range where disturbances and setpoints important, and $|L|$ small for higher frequencies,

$$|L| >> 1, \omega \in [0, \omega_B]$$
$$|L| << 1, \omega > \omega_B$$

i.e., want $|L|$ to drop of steeply around $\omega_B \approx \omega_c$

But, slope of $|L|$ and phase arg L coupled

Bode Relationship

$$\arg L(j\omega_0) \le \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{d\ ln\,|L|}{d\ ln\,\omega} ln \left|\frac{\omega+\omega_0}{\omega-\omega_0}\right| \frac{d\omega}{\omega}$$

- equality for minimum phase systems
- with slope $N = d\ ln\,|L|/d\ ln\,\omega$,

$$\arg L(j\omega_0) \approx \frac{\pi}{2} N(j\omega_0)$$

Thus, slope around crossover ω_c should be at most -2, less to yield some phase margin.

A Procedure for Loop Shaping

1. First try

$$L(s) = \frac{\omega_c}{s} \quad \Rightarrow \quad K = \frac{\omega_c}{s} G^{-1}(s)$$

   yields

$$y = \frac{\omega_c}{s + \omega_c} r$$

   But, bad disturbance rejection if G_d slow

2. For disturbances

$$e = SG_d d$$

   Require

$$|SG_d| < 1 \quad \forall\omega$$

   corresponds to

$$|1 + L| > |G_d| \quad \forall\omega$$

A Procedure for Loop Shaping

cont. If $|G_d| > 1$ we get approximately

$$|L| > |G_d|$$

Simple choice

$$L = G_d \quad \Rightarrow \quad K = G^{-1} G_d$$

and with integral action

$$K = \frac{s + \omega_I}{s} G^{-1} G_d$$

3. High frequency correction

$$K = \frac{s + \omega_I}{s} G^{-1} G_d \frac{\tau s + 1}{\frac{\tau}{\gamma} s + 1}$$

to improve stability, i.e., modify slope of $|L|$ around ω_c

4. To improve setpoint tracking, add prefilter $K_r(s)$ on setpoint $\Rightarrow$ 2-DOF control structure

Shaping the Closed Loop

- Shaping $L = GK$ is just a means of achieveing a desired closed-loop
- Alternative: find controller that minimizes a weighted sensitivity, e.g.,

$$\min_K \left(\max_\omega |w_P S| \right) = \min_K \|w_P S\|_\infty$$

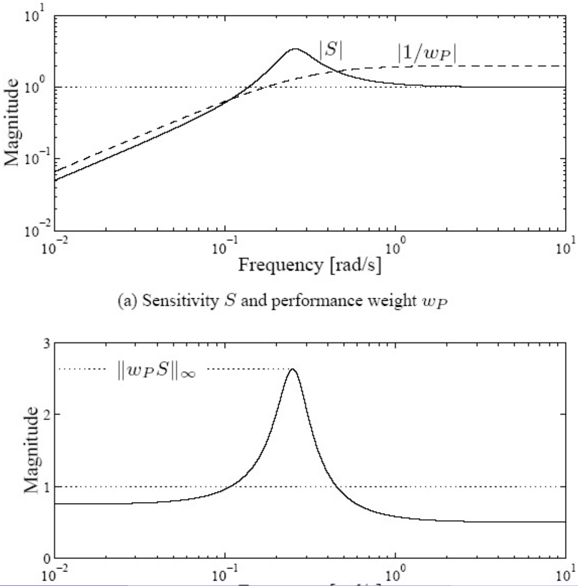
  - $\|wpS\|_\infty < 1 \quad \Rightarrow \quad |S| < 1/|w_p| \quad \forall \omega$

- Remark: $\|\cdot\|_\infty$ is called the $\mathcal{H}_\infty$-norm, ∞ because

$$\max_\omega |F(j\omega)| = \lim_{p \to \infty} \left(\int_{-\infty}^{\infty} |F(j\omega)|^p d\omega \right)^{1/p}$$

and $\mathcal{H}$ denotes *Hardy space*. Here, $\mathcal{H}_\infty$ is the set of all stable and proper transfer-functions.

Weighted Sensitivity

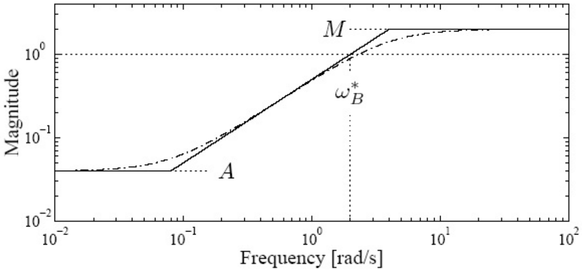


Performance weight

Typical choice for weigth

$$w_p = \frac{s/M + \omega_B}{s + \omega_B A}$$

Magnitude of $1/|w_p|$:



Control objective satisfied if $\|w_p S\|_\infty < 1$

Next Time

- Fundamental performance limitations and tradeoffs in SISO feedback
- *Controllability analysis:* what is achieveable performance, e.g., ω_B, M, for a given system?

Homework:

- Exercise 1 (download from the course homepage). Hand in next Friday.
- Read Chapter 5 (and 1-2) in Skogestad and Postlethwaite

First Exercise

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Exercise 1 – SISO loop-shaping

Given a plant model $y(s) = G(s)u + G_d(s)d$ with

$$G(s) = \frac{5}{(5s+1)(0.5s+1)^2} \; ; \quad G_d(s) = \frac{2}{5s+1}$$

Design a feedback controller, e.g., using loop-shaping ideas, that satisfies the following objectives:

1. *Tracking:* rise time $< 1s$, overshoot $< 5\%$.
2. *Rejection of unit step disturbance:* $|y| < 0.1$ at all times, $|y| < 0.01$ after $5s$.
3. *Input constraints:* $|u| < 2$ at all times.

   Plot the resulting sensitivity, complementary sensitvity, disturbance sensitivity and loop-gain as functions of frequency, and verify the performance through simulations in the time domain.

   Tip: scale the problem and determine the approximate requirements on the closed-loop

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